

Modelling Magnetoconvection in Sunspots

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Preface

All work described in this dissertation is believed to be original except where explicit reference has been made to other authors. This dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration except where specifically indicated in the text. No part of this thesis has been submitted for any qualification other than the degree of Doctor of Philosophy at the University of Cambridge.

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Summary

Although sunspots have been observed for centuries, we do not yet have a detailed understanding of their structure and properties. This thesis considers a number of approaches to modelling sunspots, and in particular the magnetoconvective processes that take place within them. Particular emphasis is given to the penumbra, since little is currently known theoretically about this region.

Our models will fall into two categories. The first category includes simplified, reduced models, covering the linear and weakly nonlinear regimes, while the second category consists of full three-dimensional numerical simulations of compressible magnetoconvection.

We begin by constructing a simple linearized model of magnetoconvection in uniform inclined fields. We then move on to the weakly nonlinear regime, where competition between different patterns of convection can be investigated. Finally we extend the model by allowing the angle of inclination of the field to vary with position. This allows us to build up a reasonable reduced model of convection in a sunspot, showing a transition between ‘umbra’ and ‘penumbra’; although this model has its limitations, it serves to show how much can be achieved using a minimal approach, without needing a detailed knowledge of the physics of sunspots.

The remainder of the thesis presents a number of three-dimensional simulations of magnetoconvection. Our simulations were designed to illustrate how the pattern of convection changes as the angle of inclination of the field is varied, with the aim of investigating the patterns that might be expected within the penumbra. We begin with simple simulations with uniform fields, and then move on to a model in which the field ‘fans out’ with position, mimicking the situation in a sunspot. The results show an interesting transition from hexagonal to more roll-like patterns, reminiscent of the transition between umbra and penumbra found in real sunspots.

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