Chapter 6

Conclusions and future work

The problem of sunspots remains far from solved. It is true that the basic physical properties and large-scale structure of sunspots can be described reasonably well by considering axisymmetric models (see also section 1.3 in the Introduction). However, the more interesting problem is to explain the fine-scale structure of sunspots, and this is far more challenging. In order to go about this, one needs a detailed understanding of magnetoconvection, and in particular the ways in which the form taken by the convection might vary within different parts of the sunspot.

One of the most fundamental variations between different parts of a sunspot is the difference between the umbra (the central region) and the penumbra (the outer region). This is presumably the result of some new form of convection appearing once the magnetic field becomes sufficiently inclined to the vertical. However, in order to make this idea more concrete, we need to produce models or simulations of magnetoconvection that demonstrate the transitions that occur as the angle of inclination of the field is varied. In this thesis, we have produced several models that do exactly this, ranging from simplified linearized models within uniform fields to three-dimensional numerical simulations involving non-uniform field configurations.

We began in Chapter 2 with an idealized, linear Boussinesq model, which showed the asymmetry between left- and right-travelling waves. In the oscillatory regime ($\zeta < 1$), different types of rolls (parallel, perpendicular and oblique) were possible as the tilt was varied, whereas in the steady regime ($\zeta > 1$), the parallel rolls were always preferred near onset.

In Chapter 3 we moved on to look at a number of weakly nonlinear models. The simplest of these looked at the steady regime ($\zeta > 1$) and considered solutions on a hexagonal lattice. This model is also perhaps the most relevant to sunspots (since compressible simulations, intended to model sunspots, have indicated that convection is steady near onset – see for example Weiss et al. 1990, 1996).

This hexagonal model allows us to investigate the transition between hexagons, for small tilt angle ϕ, and field-aligned rolls, for larger ϕ; this transition has been known about for some time (Danielson, 1961), and is likely to be responsible (at least in part) for the difference in appearance between umbra and penumbra. Our model has shown that this transition can be associated with hysteresis.

In Chapter 5 we presented a number of numerical simulations of compressible magnetoconvection within inclined fields. The results (Figure 5.2, page 161) may be compared with the predictions of the weakly nonlinear model (Figure 3.9b, page 83). The transition between hexagons and rolls was observed, as was the lengthening of the hexagonal cells in the direction of tilt for larger ϕ, as predicted. A new pattern of 'wavy' cells was also found, as well as turbulent solutions for larger R. However, we did not find anything analogous to the hysteresis observed in the weakly nonlinear model. (Of course, this does not prove that the hysteresis is not present, since it may be that it is found only at parameter values that were omitted from our investigation.)

One aspect that we did not investigate is the dependence of these results on the value of Q (which measures the field strength). Our results have mostly concentrated on the strong-field regime, where the anisotropy has a strong effect, so that the patterns tend to align themselves with the field. However, when Q is small or zero, we would of course expect isotropic patterns. It would be interesting to investigate what happens for intermediate values of Q, since this would reveal the details of the transition between these two extremes.

The remaining work in this thesis has been concerned with overall sunspot structure, and the transition between umbral and penumbral forms of convection, which we have studied by using models in which the magnetic field inclination is a function of position. In Chapter 4 this was done within the context of a weakly nonlinear model, while in Chapter 5 full compressible simulations were attempted.

Chapter 4 demonstrated that a fairly simple model, using only the basic symmetries of the problem and no specific knowledge of solar physics, can nevertheless reproduce many of the basic properties of sunspots. Of particular note is the relatively sharp transition from hexagonal convection (in the umbra) to roll-like convection (in the penumbra) found in the model. This demonstrates that the umbra-penumbra transition (viewed as a transition from one type of convection to another) is a fundamental property of this kind of convection, and does not rely on solar physics as such, but simply the (minimal) assumptions regarding symmetries of the situation. The model does not however produce anything resembling the complex filamentary structure found in real sunspots. This can be interpreted as saying that the filamentary structure is less fundamental and *does* require more complex solar physics for its understanding.

Finally, in Chapter 5 we aimed to reproduce these kinds of results within a fully non-linear, compressible simulation. Here, we set up a magnetic field varying from vertical at one end of the domain to horizontal at the other, and then ran the simulation to see what form of convection would be produced. The main result of these simulations was that a relatively sharp transition between two different forms of convection, one in the umbra and one in the penumbra, was found.

One problem with this model was that there was, within our chosen initial field setup, quite a large contrast in the field strength between the umbra and penumbra. We typically found that either the field within the umbra was too strong (preventing any umbral convection from occurring) or that the field within the penumbra was too weak (meaning that the field did not exert a very strong influence on the resulting convection pattern). Another issue was that we were forced to use a perfectly conducting upper boundary condition (while a potential field condition would have been more realistic) in order to prevent flux from escaping via magnetic buoyancy. We hope that future work will be able to produce an improved model that solves some of these problems, and also to investigate the various convective regimes in more detail than we were able to here.

In conclusion, this thesis has looked at different ways of modelling the changes in the form of magnetoconvection that would be expected as the tilt angle of the magnetic field (ϕ) is varied. In particular, we conclude that a sharp transition in the form of convection, from hexagons to more roll-like structures, would be expected once ϕ exceeds some critical value (as indicated by the results of Chapters 4 and 5); this transition is a robust effect, and is therefore likely to be at least partly responsible for the distinction between the umbra and the penumbra of a sunspot.

We say only 'partly responsible' because a change in the form of convection would not in itself be enough to account for all of the rich and complex structure that is observed in real penumbrae. Rather, the transition from hexagonal to roll-like convection is likely to act as a starting point which triggers the formation of the more complex magnetic structures and flow patterns that make up a real penumbra. We hope that the models presented here (and especially the compressible models of Chapter 5) will provide a suitable starting point for more detailed investigations into these enigmatic phenomena.